How Did Price's Metonic Cycle Gear Train Work?

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1 Introduction

The advent of new insight into the structure and function of the Antikythera Mechanism [9] made me wonder exactly how Derek J. de Solla Price's [13] proposed Metonic Cycle gear train in the Mechanism works. I decided to look at the Metonic gear train first since it is a simple gear train; specifically this gear train has no epicyclic gears [17] or pin-and-slot devices [1, 4].

I'll just note here that while Price's Metonic Cycle gear train is generally considered to be correct [7], his interpretation of its output (and hence the upper dial on the back of the Mechanism) as well as how the Metonic Cycle pointer was turned, is considered to be wrong [5].

These notes briefly investigate how and why Price's Metonic Cycle gear train works.

2 First, What Is The Metonic Cycle?

The Metonic Cycle is *a period of approximately nineteen years after which the phases of the moon recur on the same day of the year*. It is defined by observation to be 235 synodic (lunar) months, just 1h27m33s longer than nineteen tropical years. Learning from the Babylonian and Hebrew lunisolar calendars in which the years 3, 6, 8, 11, 14, 17, and 19 are the long (13-month) years, the 5^{th}century BC Greek mathematician, astronomer, geometer, and engineer Meton of Athens [14] judged the cycle to be a whole number of days, specifically 6,940 days. Using these integer values facilitated the construction of a lunisolar calendar.

One Metonic Cycle is defined to be 19 tropical years, which is 235 synodic months (lunar phases), which in turn equals 6,939.688 days. Since 19 tropical years equals 6,939.602 days the difference of $6,939.688 - 6,939.602 = 0.086$ days/cycle means that after twelve cycles there will be a 1.032 day difference between observation and calculation (since 0.086 days/cycle $* 12$ cycles $= 1.032$ days).

The Metonic Cycle also turns out to be very close to integer multiples of two other important lunar periods:

- 254 sidereal months (lunar orbits) = 6,939.702 days
- 255 draconic months (lunar nodes) = 6,939.116 days

So in summary:

One Metonic Cycle	= 19 tropical years	# 6,939.602 days
	≈ 235 synodic months	# 6,939.688 days
	≈ 254 sidereal months	# 6,939.702 days
	≈ 255 draconic months	# 6,939.116 days

Interestingly $\frac{254}{19} \approx 13.36842$, which is said to be an important astronomical constant[1].

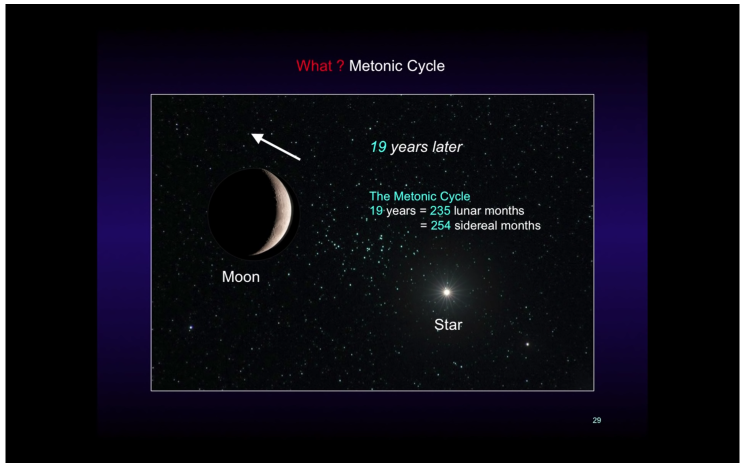

Figure 1: The Metonic Cycle [8]

This is all very interesting. However, the Metonic Cycle seems to be a coincidence. The periods of the Moon's orbit around the Earth and the Earth's orbit around the Sun are believed to be independent, and not to have any known physical resonance. An example of a non-coincidental cycle is the orbit of Mercury, with its 3:2 spin-orbit resonance [2].

3 Price's Metonic Cycle Gearing Scheme

Price's Metonic gearing scheme, described in his classic work "Gears from the Greeks. The Antikythera Mechanism: A Calendar Computer from ca. 80 B. C. T" [3], is shown in Figure 2. As we will see below, Price's Metonic Cycle gear train essentially calculates the average position of the Moon in the Zodiac.

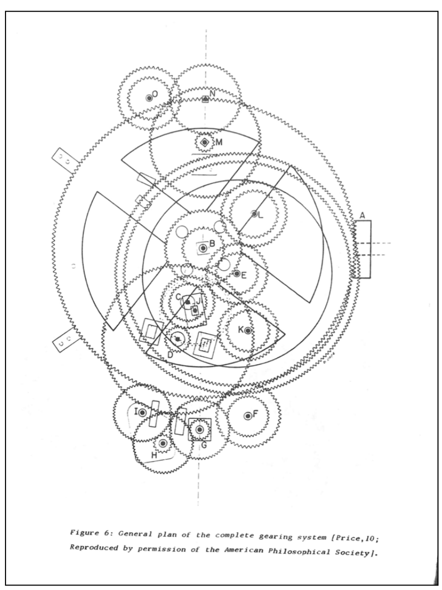

Figure 2: Price's General Gearing Plan [3]

[1]Why exactly this constant is considered to be "important" is something I have not been able to learn.

For calculating gear ratios, Price's sectional gearing diagram is more useful. As we can see from Figure 3, the gears of interest are b2, c1, c2, d1, d2, and e2, with the following tooth counts[2]:

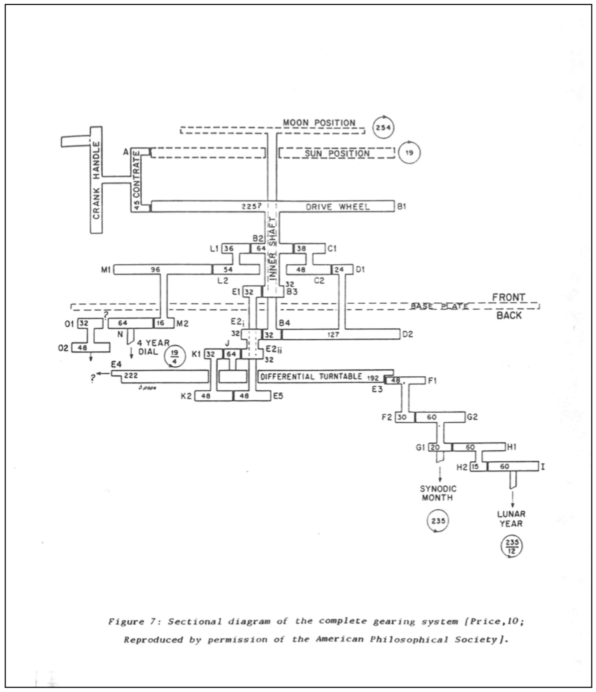

Figure 3: Price's Sectional Gearing Diagram [3]

- b2: 64 teeth
- c1: 38 teeth
- c2: 48 teeth
- d1: 24 teeth
- d2: 127 teeth
- e2: 32 teeth

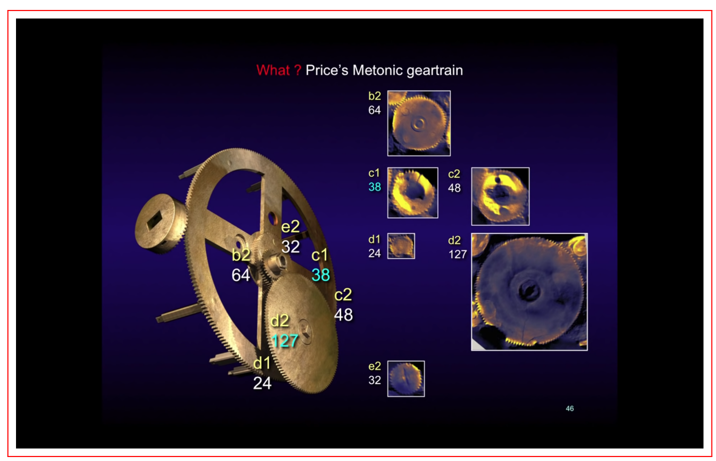

Figure 4: The Metonic Gear Train [8]

We know that in simple gear trains we can calculate the Gear Ratio (GR) as

$$\mathrm{GR} = \frac{\text{Number of Teeth on the Driven Gear}}{\text{Number of Teeth on the Driver Gear}}$$

[2]Price argued with Greek physicist Charalambos Karakalos about tooth counts on various gears [8].

In addition, we know that the driven gear rotates in the opposite direction of the driver gear.

With this information we can start to calculate what Price's Metonic gear train does.

Specifically:

$$
\begin{array}{lll}
\frac{\mathrm{b2}}{\mathrm{c1}} & = -\frac{64}{38} = -\frac{32}{19} & \text{\# driver \& driven gears turn in opposite directions} \\
\frac{\mathrm{b2}}{\mathrm{c1}} \times \frac{\mathrm{c2}}{\mathrm{d1}} & = -\frac{64}{38} \times -\frac{48}{24} = -\frac{32}{19} \times -\frac{2}{1} = \frac{64}{19} & \text{\# } \frac{\mathrm{c2}}{\mathrm{d1}} \text{ multiplies } \frac{\mathrm{b2}}{\mathrm{c1}} \text{ by 2} \\
\frac{\mathrm{b2}}{\mathrm{c1}} \times \frac{\mathrm{c2}}{\mathrm{d1}} \times -\frac{\mathrm{d2}}{\mathrm{e2}} & = -\frac{64}{38} \times -\frac{48}{24} \times -\frac{127}{32} = -\frac{254}{19} & \text{\# } \frac{254}{19} \approx 13.36842 \text{ sidereal months/year}
\end{array}
$$

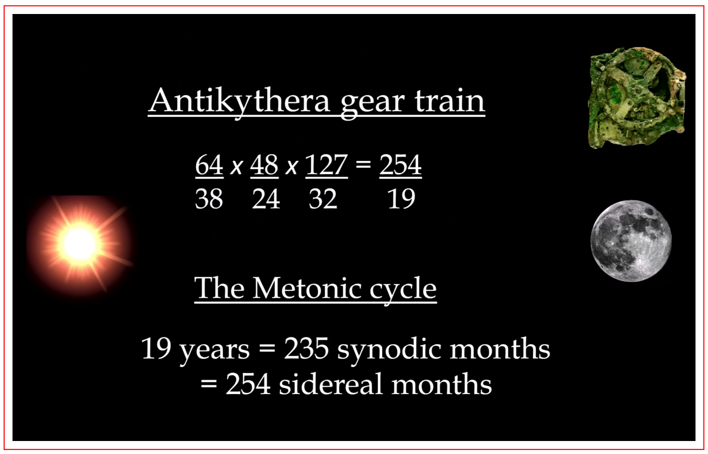

Figure 5: Metonic Gear Train Ratios and the Metonic Cycle

4 Putting It All Together

The Antikythera Mechanism is thought to have been operated by a knob or crank on the side of the device. This knob (or crank) was connected to a crown gear that meshed with b1, the main drive gear. b1 is the large, four spoked gear seen in Fragment A (see Figure 6), and one revolution of b1 represents one year[3]. Since b2 is planted on b1 to form a compound gear (b1 and b2 are connected to the same axle; see Figures 3 and 6), one revolution of b2 also represents one year.

[3]For this reason b1 is sometimes called the "solar gear" [1].

Figure 6: Fragment A of the Antikythera Mechanism [12]

This configuration of gears means that one revolution of b2 (or b1) moves the Metonic pointer by one nineteenth of the Metonic Cycle, or 13.36842 sidereal months. Thus 19 revolutions of the main drive gear results in one revolution of the Metonic pointer or one Metonic Cycle, just as required.

Unfortunately Price was wrong. He incorrectly thought that the output of his Metonic Cycle gear train, at the 32 tooth gear e2 (this gear also has an interesting pentagonal hub), went directly to the Zodiac dial on the front of the Mechanism. This is the dial that Albert Rehm [11] had proposed to show the average position of the Moon in the Zodiac.

Looking ahead a bit, what Price's Metonic gear train actually does is to calculate the mean sidereal month. This output, at e2, is then used to drive the 50 tooth gear e5, which is understood to rotate once per sidereal month [10]. e5 is part of the four gear pin-and-slot device, who's output is the variable sidereal month. The pin-and-slot device is mounted epicyclically on gear e3, which changes its period of variation from the variable sidereal month to the anomalistic month. A summary of this amazing gearing is shown in Figure 7.

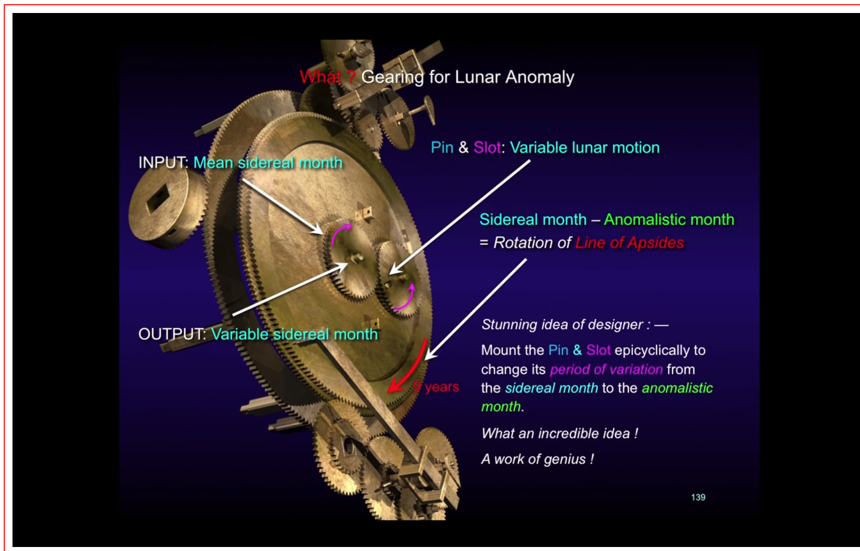

Figure 7: Gearing For The Lunar Anomaly [8]

5 Michael Wright's Scheme For Turning The Metonic Pointer

In Price's model the Metonic Cycle gear train output directly to a pointer on the proposed Zodiac dial on the front of the Mechanism. Michael Wright suggested a different scheme for the Metonic dial and pointer [16]. Wright noted Price's comment that the back upper dial, the Metonic dial, might be a five turn dial since one Metonic cycle equals 235 synodic months which equals 5×47 lunar months [3]. As a result Wright proposed that the Metonic dial was a five turn spiral dial in which each revolution represented 47 lunar months and proposed the following gearing scheme to move the Metonic pointer on this five turn dial:

- l1: 38 teeth
- b2: 64 teeth
- l2: 53 teeth
- m1: 96 teeth
- m2: 15 teeth
- n1: 53 teeth

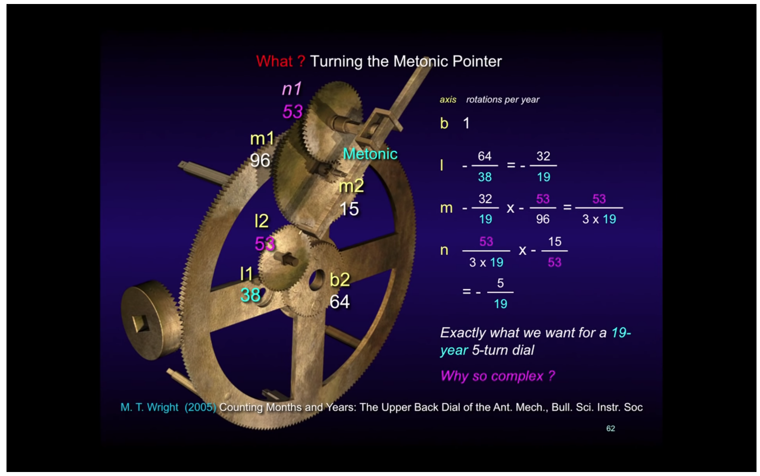

Figure 8: Wright's Metonic Gear Train [8]

Wright's gear ratios work out like this:

$$-\frac{\text{b2}}{\text{l1}} = -\frac{64}{38} = -\frac{32}{19}$$

$$-\frac{\text{b2}}{\text{l1}} \times -\frac{\text{l2}}{\text{m1}} - = -\frac{64}{38} \times -\frac{53}{96} = -\frac{32}{19} - \times - \frac{53}{96} = \frac{53}{3\times 19}$$

$$-\frac{\text{b2}}{\text{l1}} \times -\frac{\text{l2}}{\text{m1}} - \times - \frac{\text{m2}}{\text{n1}} = -\frac{64}{38} \times - - \frac{53}{96} \times -\frac{15}{53} = -\frac{32}{19} \times - - \frac{53}{3\times 32} \times -\frac{3\times 5}{53} = - - \frac{5}{19}$$

So one revolution of the main drive gear (b1) moves Wright's Metonic pointer by $\frac{5}{19} \approx 0.2632$, and nineteen revolutions of b1 (one Metonic Cycle) results in five revolutions of Wright's Metonic pointer, consistent with Wright's five turn spiral Metonic dial model.

BTW, note that the two 53 tooth gears l2 and n1 cancel in this gear train, so why are they there? Part of the answer is to drive the Saros eclipse prediction dial [6], a subject for another note. However, for reference: one Saros Cycle after an eclipse the Sun, Earth, and Moon return to approximately the same relative geometry (close to a straight line) and a nearly identical eclipse will occur in what is referred to as an eclipse cycle [15].

It turns out that the Saros Cycle is a remarkable chance resonance between three of the orbital cycles of the moon: the synodic, draconic and anomalistic lunar cycles.

$$
\begin{aligned}
\text{One Saros Cycle} &= 223 \text{ synodic months} \\
&\approx 242 \text{ draconic months} && \# \ 241.999 \text{ draconic months} \\
&\approx 239 \text{ anomalistic months} && \# \ 238.992 \text{ anomalistic months} \\
&\approx 19 \text{ eclipse years (38 eclipse seasons)} && \# \ 18.999 \text{ eclipse years} \\
&= 6{,}585.321347 \text{ solar days} \\
&= 18.029 \text{ tropical years}
\end{aligned}
$$

6 Conclusions

Acknowledgements

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LaTeX Source

https://www.overleaf.com/read/ndpvkytkhmbv

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