A Few Notes on Special Relativity

David Meyer

dmm613@gmail.com

Last Update: December 24, 2024

Initial Version: December 23, 2024

1 Introduction

TBD

2 The Postulates of Special Relativity

1. The laws of physics are the same for all observers in uniform motion relative to each other (i.e., in inertial reference frames). This means that no inertial observer can detect a preferred or absolute state of motion. There is no absolute ”rest” frame; all motion is relative.

2. The speed of light in a vacuum is always constant, regardless of the motion of the observer or the light source. This speed is approximately $c = 299,792,458$ m/s.

3 Time Dilation

Time dilation is an unusual phenomenon predicted by Einstein's special theory of relativity. It refers to the effect where time appears to pass more slowly for an observer in motion relative to a stationary observer. In simpler terms, a moving clock ticks more slowly compared to a clock that is at rest, as observed by someone who is not moving with the clock [1].

The key idea is that when an object moves at a speed close to the speed of light relative to an observer, time for the moving object (as seen by the observer) slows down. This effect becomes more pronounced as the speed of the object approaches the speed of light.

Said another way, special relativity tells us that for an observer O in an inertial frame of reference, a clock that is moving relative to the observer will be measured to tick more slowly than a clock at rest in the observer's frame of reference [3].

To see why this happens, consider the following thought experiment. The experimental setup is shown in Figure 1 [1]. Here the observer, light source and detector are at rest in inertial frame of reference I (even though the train car is moving with velocity v).

[1] Assume here that the thickness of the source and the detector are negligible so that the distance light travels is h.

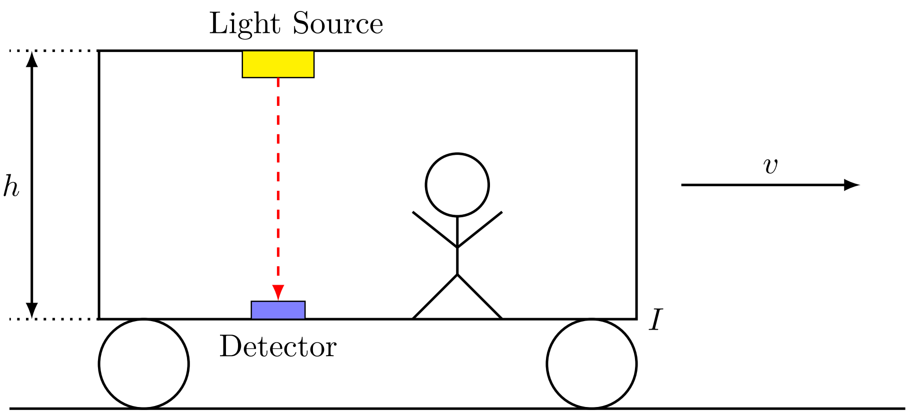

Figure 1: Observer at rest with respect to inertial frame of reference I

For observer at rest with respect to an inertial frame of reference, the time taken for a beam of light to travel from the light source to the detector is

$$\begin{aligned} h &= c\Delta t_0 && \text{\# } \Delta t_0 \text{ is called the proper time [2]} \\ \Rightarrow & \quad \Delta t_0 = \frac{h}{c} && \text{\# solve for } \Delta t_0 \end{aligned}$$

Now consider the same experimental setup as in Figure 1 except this time the observer is on the train landing. Call this inertial frame of reference I'.

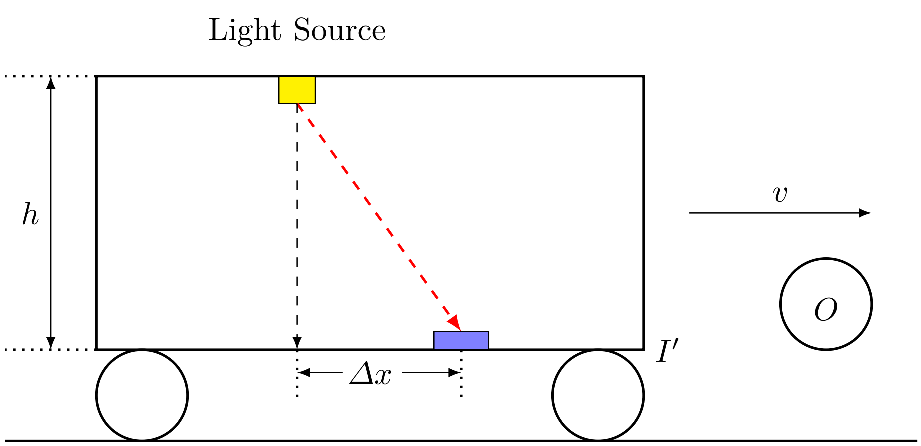

Figure 2: Observer at rest with respect to another (moving) inertial frame of reference

The question now is how much time, call it Δt, passes from the point of view of observer O when the light travels from the source to the detector. Δx is the distance the train moves forward in time Δt. We also know that the speed of light in a vaccum is c for all observers, so distance that the light travels in this case is $c\Delta t$ (the hypotenuse of the triangle show in Figure 2).

The h, Δx, $c\Delta t$ triangle is abstracted in Figure 3.

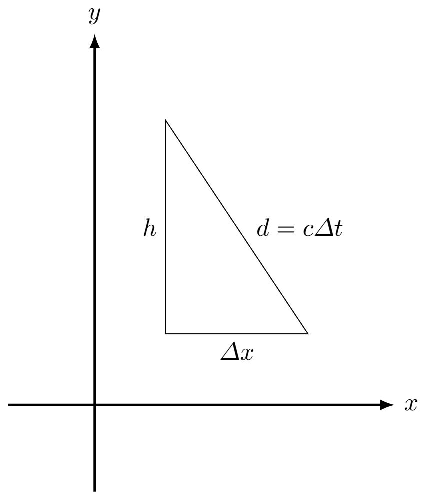

Figure 3: Triangle formed by the motion of I' relative to I

Using this triangle we can calculate Δt using the Pythagorean theorem:

$$
\begin{aligned}
(c\Delta t)^2 &= h^2 + (\Delta x)^2 && \text{\# by the Pythagorean theorem} \\
&\Rightarrow (c\Delta t)^2 = (c\Delta t_0)^2 + (\Delta x)^2 && \text{\# } h = c\Delta t_0 \\
&\Rightarrow (c\Delta t)^2 = (c\Delta t_0)^2 + (v\Delta t)^2 && \text{\# velocity } v \text{ of the train: } v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v\Delta t \\
&\Rightarrow c^2(\Delta t)^2 = c^2(\Delta t_0)^2 + v^2(\Delta t)^2 && \text{\# square terms} \\
&\Rightarrow (\Delta t)^2 = (\Delta t_0)^2 + \frac{v^2}{c^2}(\Delta t)^2 && \text{\# divide both sides by } c^2 \\
&\Rightarrow (\Delta t)^2 - \frac{v^2}{c^2}(\Delta t)^2 = (\Delta t_0)^2 && \text{\# subtract } \frac{v^2}{c^2}(\Delta t)^2 \text{ from both sides} \\
&\Rightarrow (\Delta t)^2 \left(1 - \frac{v^2}{c^2}\right) = (\Delta t_0)^2 && \text{\# factor out } (\Delta t)^2 \\
&\Rightarrow (\Delta t)^2 = \frac{(\Delta t_0)^2}{\left(1 - \frac{v^2}{c^2}\right)} && \text{\# divide both sides by } \left(1 - \frac{v^2}{c^2}\right) \\
&\Rightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} && \text{\# take the square root of both sides}
\end{aligned}
$$

So for observer O the passage of time on the moving train slows down by a factor of

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

Equation (1) is called the Lorentz factor (or sometimes the gamma factor). Note also that Δt can be written using the Lorentz factor: $\Delta t = \gamma \Delta t_0$.

4 Length Contraction

5 Conclusions

Acknowledgements

LaTeX Source

https://www.overleaf.com/read/xfghcdxjyzhg#675841

References

[1] Scott Hughes. Spacetime: Introduction to Special Relativity. https://web.mit.edu/sahughes/www/8.022/lec11.pdf, 2005. [Online; accessed 24-December-2024].

[2] Wikipedia contributors. Proper time — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Proper_time&oldid=1238644995, 2024. [Online; accessed 24-December-2024].

[3] Wikipedia contributors. Time dilation — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Time_dilation&oldid=1260859390, 2024. [Online; accessed 24-December-2024].