# Is $\sqrt[i]{i}$ a Real Number?

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Well, consider

$$
\begin{aligned}
\sqrt[i]{i} &= i^{\frac{1}{i}} && \# \ \sqrt[n]{x} = x^{\frac{1}{n}} \\
&= i^{-i} && \# \ x^{\frac{1}{n}} = x^{-n} \\
&= \frac{1}{i^i} && \# \ x^{-n} = \frac{1}{x^n} \\
&= \frac{1}{e^{\left(\frac{i\pi}{2}\right)^i}} && \# \ i = e^{\frac{i\pi}{2}} \ [1] \\
&= \frac{1}{e^{\frac{i^2\pi}{2}}} && \# \ (x^m)^n = x^{mn} \\
&= \frac{1}{e^{-\frac{\pi}{2}}} && \# \ i^2 = -1 \\
&= e^{\frac{\pi}{2}} && \# \ \frac{1}{x^{-n}} = x^n \\
&\approx 4.8105 && \# \ e^{\frac{\pi}{2}} \in \mathbb{R} \Rightarrow \sqrt[i]{i} \in \mathbb{R}
\end{aligned}
$$

So apparently $\sqrt[i]{i}$ is a real number and in particular $\sqrt[i]{i} = e^{\frac{\pi}{2}} \approx 4.8105$.

## References

[1] David Meyer. Is i to the i a real number and if so, what does it equal? `https://davidmeyer.github.io/qc/i_to_the_i_is_real.pdf`, 2021. [Online; accessed 09-June-2022].