Notice that $\sum\limits_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}$. OK, but why? Consider

$$
\begin{aligned}
S &= \sum_{n=1}^{\infty} \frac{1}{3^n} && \text{\# define } S \\
&= \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots && \text{\# expand terms} \\
\Rightarrow\ & 3 \cdot S = 3 \cdot \left[\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\right] && \text{\# multiply both sides by 3} \\
\Rightarrow\ & 3 \cdot S = 1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots && \text{\# multiply through on right side} \\
\Rightarrow\ & 3 \cdot S = 1 + S && \text{\# definition of } S \\
\Rightarrow\ & 3 \cdot S - S = 1 && \text{\# subtract } S \text{ from both sides} \\
\Rightarrow\ & 2 \cdot S = 1 && \text{\# } 3 \cdot S - S = 2 \cdot S \\
\Rightarrow\ & S = \frac{1}{2} && \text{\# } S = \sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}
\end{aligned}
$$