

# The Spiral of Theodorus

David Meyer

dmm613@gmail.com

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## 1 A Bit About The Spiral

The Spiral of Theodorus [2] is the fantastic object shown in Figure 1. It is composed of right triangles placed edge-to-edge, where the sides of  $n^{\text{th}}$  triangle are of length 1 and  $\sqrt{n}$  and the hypotenuse is of length  $\sqrt{n+1}$ .

The spiral is named after Theodorus of Cyrene, the ancient Greek mathematician who lived during the 5<sup>th</sup> century BC and who apparently discovered the spiral [3]. Theodorus used the spiral to prove that all of the non-square integers in the interval  $[3, 17]$  are irrational [2]. Theodorus' original spiral stopped at  $\sqrt{17}$ , apparently because  $\sqrt{17}$  is the length of the hypotenuse of the last triangle in the spiral that doesn't overlap; this is shown in Figure 1.

Interestingly, in 1958 Erich Teuffel showed that no two hypotenuses of the spiral will coincide regardless of how far the spiral is continued [1]. Said another way, no two hypotenuses of the spiral will coincide, no matter how big  $n$  gets.

In any event, the spiral is an incredible object.

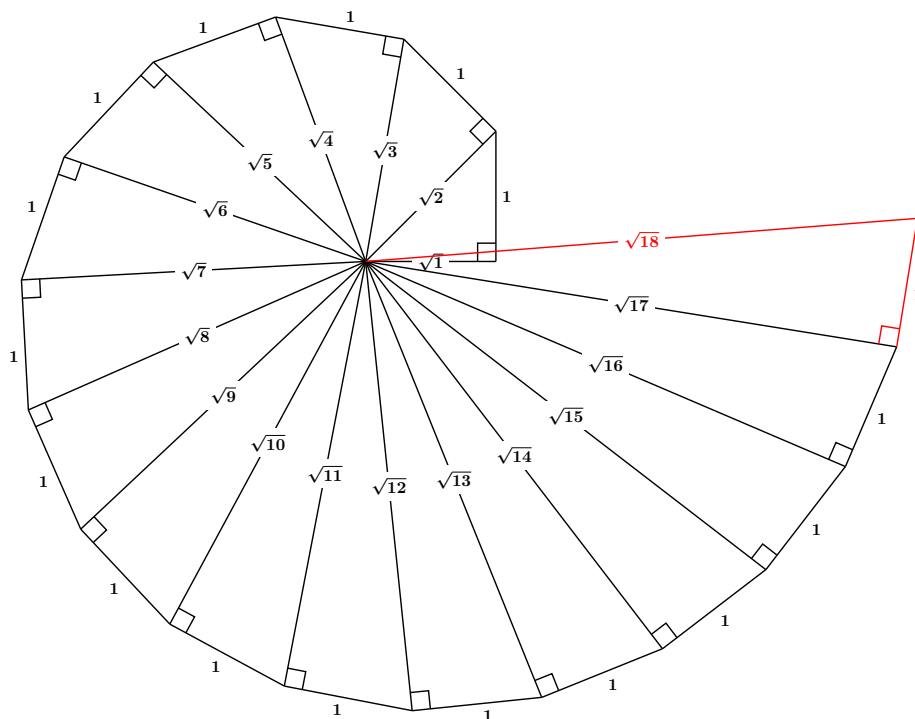


Figure 1: The Spiral of Theodorus ( $n = 17$ )

## 2 A Spiral of Theodorus with $n = 128$

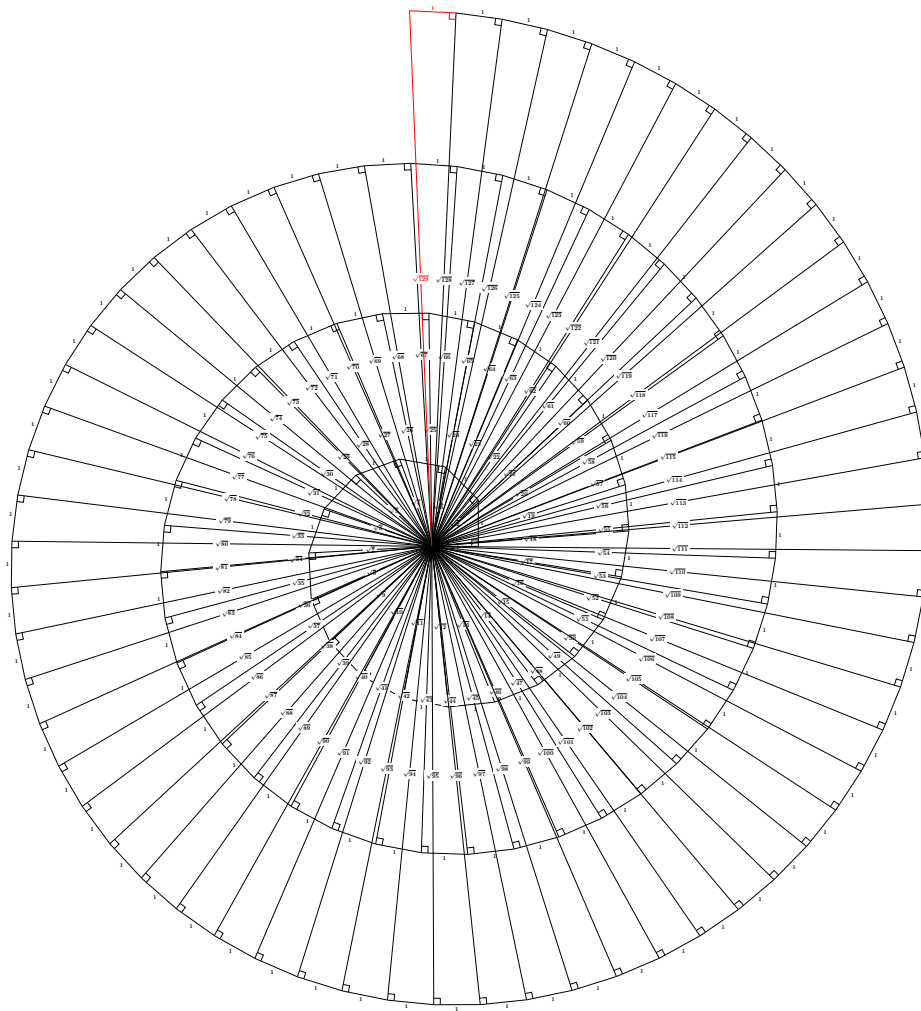


Figure 2: A spiral with  $n = 128$

### Acknowledgements

### L<sup>A</sup>T<sub>E</sub>X Source

<https://www.overleaf.com/read/scsptwzqxzdt>

### References

- [1] <https://dewiki.de>. Wurzelschnecke. <https://dewiki.de/Lexikon/Wurzelschnecke>, 2023. [Online; accessed 21-Aug-2023].
- [2] Wikipedia Contributors. Spiral of Theodorus — Wikipedia, The Free Encyclopedia. [https://en.wikipedia.org/w/index.php?title=Spiral\\_of\\_Theodorus&oldid=1091008882](https://en.wikipedia.org/w/index.php?title=Spiral_of_Theodorus&oldid=1091008882), 2022. [Online; accessed 17-August-2022].

- [3] Wikipedia Contributors. Theodorus of Cyrene — Wikipedia, The Free Encyclopedia. [https://en.wikipedia.org/w/index.php?title=Theodorus\\_of\\_Cyrene&oldid=1082503450](https://en.wikipedia.org/w/index.php?title=Theodorus_of_Cyrene&oldid=1082503450), 2022. [Online; accessed 17-August-2022].