

# A Bit on Vector Spaces in Quantum Mechanics

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## 1 What is a State in Quantum Mechanics?

Hilbert space  $\mathcal{H}$ .

## 2 The Real Projective Line

The real projective line is the set of all lines that pass through the origin. One way to think about this is as the one-dimensional subspace of "rays" [1, 2]. This is shown on the left in Figure 1 (or Appendix A).

If we slide the red point along the  $y = 1$  line in Figure 1 the angle  $\theta$  varies between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$  as  $x$  varies between  $-\infty$  and  $\infty$ . In particular, as  $x \rightarrow \pm\infty$ ,  $\theta \rightarrow \pm\frac{\pi}{2}$ . That is

$$\lim_{x \rightarrow \infty} \theta = \frac{\pi}{2}$$

and

$$\lim_{x \rightarrow -\infty} \theta = -\frac{\pi}{2}$$

If we then look at the points  $(x, \theta)$  you find that the real projective line can be seen as a circle. This is shown on the right in Figure 1.

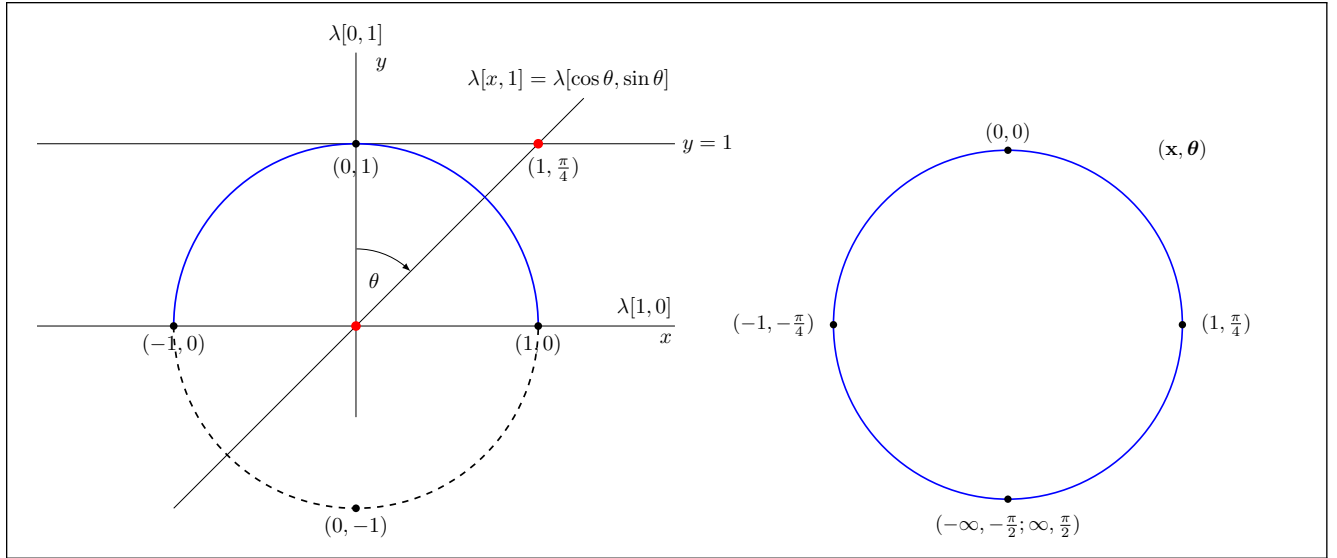


Figure 1: Real Projective Line Setup

### 3 The Real Projective Plane

#### Acknowledgements

#### L<sup>A</sup>T<sub>E</sub>X Source

<https://www.overleaf.com/read/gjysgsdftjxy>

#### References

- [1] Gabriele Carcassi. Understanding vector spaces in quantum mechanics. <https://www.youtube.com/watch?v=KEzmw6cK01U>, 2023. [Online; accessed 22-Jan-2024].
- [2] Weisstein, Eric W. Ray. <https://mathworld.wolfram.com/Ray.html>, 2024. [Online; accessed 13-March-2024].

# Appendix A: The Real Projective Line

